CANTO I.

Translation.

1. In this earth there once was a great city of the name of Ayodhyá; a city that surpassed all other cities in respect of wealth and prosperity. So prosperous (was it, that it looked) as if it had fallen down from heaven by the weight of its great wealth. It was a city which was a great resort of the Kshattriya race, as the S'amí tree is the constant abode of fire.

2. The moon became radiant by the reflected refulgence of the rubies that decked the spires of the lofty edifices of that city. Nay, her (tho moon's) countenance became florid through jealous wrath at the sight of the superior charms of the fair females that lived there.

3. The opulence and prosperity of that city brought joy to all, except to young maidens that sought their lovers. For the lustre that issued from the gems of the golden gates of that city dissipated darkness and made night bright as day.

4. The glowing flags of China satin, which streamed in the sky from the lofty steeples of the mansions of that city, seemed like projections chiselled out from the moon.

5. The swans that were swimming in the moat surrounding the city-wall cast wistful looks towards tho lakes of the city; but out of despair, owing to the lofty walls which stood in their way, they were reminded of the exploits of Paraśu-ráma, who by his arrow cut a passago through the Mount of Krauñcha.

A brief account of Bháskara, and of the works written, and discoveries made, by him.—By the late Paṇḍit Bápu Deva S'ástrí, C.I.E.

[Note by Editor.—The following paper was found amongst the papers of the deceased Paṇḍit after his death in 1890 and communicated to the Society, of which ho was an Honorary Member, by his relations. It forms a portion of tho preface to his revised edition of Mr. Wilkinson's translation of the *Goládhyáya* of the *Siddhánta S'iromaṇi*, published in the "Bibliotheca Indica," so far back as 1861. This preface was, apparently by an accident, not printed at the time, and tho Paṇḍit kept it by him, and spent considerable pains over numerous and careful corrections, which he subsequently added. There seems to be no doubt that he intended to publish it on somo future occasion, and there cannot be a better place for its appearance than the *Journal* of the Socioty of which he was so long a valued membor.]

Bháskara was born in 1036 of the *Sálivāhana* era—or in the year 1114, A. D.—Some authors mention that he was an inhabitant of Bira, a Maráṭhá villago; but he himself states, at the end of his *Goládhyáya*, that his native place was near the Sahyádri, or tho Western Gháṭs, and it appears to me that he was an inhabitant of Víjapura, the ancient metropolis of the Karnatik. Some say that he was a Maráṭhá Bráhman follower of the Yajurveda; but his method of annotating, which is still current in the Karnatik in annotating poetical works, shews that he was a Kanaṛá Bráhman of Víjapura. His father, named Maheśvara, was a very great Paṇḍit and Astronomer, and a virtuous man. He had acquired the title of *A'chárya* (Doctor) in the assembly of the Paṇḍits.

Bháskara studied all the sciences acquired by him with his father. It cannot be ascertained whether he or his father was patronized by any Rájá, or whether he was a rich or poor man. But it is certainly true that he was expert in science, a very great poet, and an excellent Astronomer.

In his time, Lalla's work on astronomy, called *S'ishya-dhívṛiddhida-Tantra*, more usually styled the *Dhívṛiddhida* simply, was much used, as the *Siddhánta-S'iromaṇi* is at present. Bháskara first made a commentary on Lalla's work, and then wrote his own work on astronomy, called *Siddhánta-S'iromaṇi*, in two parts, *Gaṇitádhyáya* and *Goládhyáya*, composing before it two introductory works: the first on Arithmetic, called *Páṭí*, or *Lílávatí*, and the second on Algebra.* He compiled his excellent work *Siddhánta-S'iromaṇi* in the 36th year of his age, or 1150, A. D. Its first part, *Gaṇitádhyáya*, is divided into 12 chapters, *viz.* :—

Chapter I. Called the *Madhyagati*, which treats of the rules for finding the mean places of the planets, contains 7 sections.

Section 1. Kinds of time.

Section 2. Revolutions of the planets, &c.

Section 3. Rules for finding the *ahargaṇa* (or enumeration of mean terrestrial days elapsed from the commencement of the Kalpa) and thence the mean places of the planets, &c.

Section 4. The dimensions of the *Brahmáṇḍa* (universe), and of the orbits of the planets, and thence the rules for finding the mean places of the planets.

Section 5. This section, called *Pratyabda-S'uddhi* (the remainders of additive months at the beginning of each year), treats of rules for finding the remainders of additive months, subtractive days, &c., at the beginning of each year, the small *ahargaṇa* (or enumeration of the days elapsed from the beginning of the current year) and thence the mean places of the planets.

Section 6. Determination of additive months and others.

Section 7. The *Deśántara* correction, &c., and conclusion of the first chapter.

* [Or *Víjagaṇita*. Both have been translated by Colebrooke,—Ed.]

Chapter II. Called the *Spashṭa-gati*, which treats of the rules for finding the apparent places of the planets.

Chapter III. Called the *Tripraśna*, treats of the rules for resolving questions on time, finding the positions of places and directions.

Chapter IV. Called *Parva-sambhava*, on the possibility of the eclipses of the sun and moon.

Chapter V. Of lunar eclipses.

Chapter VI. Of solar eclipses.

Chapter VII. Rules for finding the lengths of the shadows reflected from the planets.

Chapter VIII. On the rising and setting of the planets.

Chapter IX. On the phases of the moon and the position of the moon's cusps.

Chapter X. On the conjunction of the planets.

Chapter XI. On the conjunction of the planets with stars.

Chapter XII. Rules for finding the time at which the declinations of the sun and moon become equal.

The second part of the *Siddhanta-S'iromaṇi*, called *Goládhyáya* is divided into 13 chapters, with an appendix. Of this part the translation is given here.

[The translation of the *Goládhyáya*, or Treatise on the Sphere, being now out of print, the following account of its contents is added for the sake of completeness :—

Chapter I. In praise of the advantages of the study of the sphere.

Chapter II. Questions on the general view of the sphere.

Chapter III. Cosmography, (including a refutation of the supposition that the earth is level).

Chapter IV. On the principles of the rules for finding the mean places of the planets.

Chapter V. On the principles on which the rules for finding the true places of the planets are grounded.

Chapter VI. On the construction of an Armillary Sphere.

Chapter VII. On the principles of the rules for resolving the questions on time, space, and directions.

Chapter VIII. The explanation of the cause of eclipses of the sun and moon.

Chapter IX. On the principles of the rules for finding the time of the rising and setting of the heavenly bodies.

Chapter X. On the cause of the phases of the moon.

Chapter XI. On the use of astronomical instruments, *viz.*, (1) the gnomon, (2) the vertical circle, (3) the *Phalaka* (invented by Bháskara), (4) the *Yashṭí*, or staff, (5) the *Dhí-yantra*, or genius-instrument, (6) the self-revolving instrument, (6) the syphon.

Chapter XII. The seasons.

Chapter XIII. Useful questions,—a collection of problems. Ed.].

In this work Bháskara has variously exposed the errors of Lalla, whose work he had formerly annotated.

We now proceed to mention the discoveries of Bháskara.

1. He discovered that the earth has the inherent property of attracting all things around it,* and

2. That portion of the equation of time which is due to the inclination of the ecliptic to the equinoctial.†

3. He found out the *tátkálika*, or instantaneous motion of the variable quantities—the planet's longitude, and the sine of the arc.

Bháskara says "the difference between the longitudes of a planet found at any time on a certain day, and at the same time on the following day, is called its rough motion during that interval of time; and its *tátkálika* motion is its exact motion."

The *tátkálika*, or instantaneous motion of a planet, is the motion which it would have in a day, had its velocity at any given instant of time remained uniform. This is clear from the meaning of the term *tátkálika*, and it is plain enough to those who are acquainted with the principles of the differential calculus, that this *tátkálika* motion can be no other than the differential of the longitude of a planet. This *tátkálika* motion is determined by Bháskara in the following manner.‡

* * * * * * * * *

Now, the term *tátkálika* applied by Bháskara to the velocity of a planet, and his method of determining it, correspond exactly to the differential of the longitude of a planet and the way for finding it. Hence it is plain that Bháskara was fully acquainted with the principle of the differential calculus.§ The subject, however, was only inci-

* [*Siddhanta-S'iromaṇi.* Chap. III, 6.—Ed.]

† [*Siddhanta-S'iromaṇi.* Chap. V, 16, 17.—Ed.]

‡ [The calculations given by the author are omitted, as they have already been published in *J. A. S., B.*, Vol. XXVII, pp. 213 and ff.—Ed.]

§ [See, however, two papers by Spottiswoode in the *Journal* of the Royal Asiatic Society, Vol. XVII, p. 222 and Vol. XX, p. 345. Mr. Spottiswoode considered that the paṇḍit had overstated his case. He added 'Bháskara undoubtedly conceived the idea of comparing the successive positions of a planet in its path, and of regarding its motion as constant during the interval, and he may be said to have had some rudimentary notion of representing the arc of a curve by means of auxiliary straight lines. But on the other hand, in the method here given, he makes no allusion to one of the most essential features of the Differential Calculus, *viz.*, the infinitesimal magnitude of the intervals of time and space therein employed. Nor indeed is anything specifically said about the fact that the method is an approximative one.

'Nevertheless, with these reservations, it must be admitted, that the penetration dentally and briefly treated of by him, and his followers, not comprehending it fully, have hitherto neglected it entirely.

4. The ancient astronomers Lalla and others say that the difference between the mean and true motion of a planet becomes nothing when the planet reaches the point of intersection of the concentric and excentric. But Bháskara, denying this, says that when the planet reaches the point where the transverse diameter of the concentric cuts the excentric, the difference of the mean and true motions becomes 0.*

For let p be the mean place of a planet at any time on a certain day, and p' that at the same time on the next day; and e and $é$ be the amounts of the equation respectively: then $p+e$ and $p'+é$ will be the true places of the planet; $\therefore p'-p+(é-e)$ will be the true motion of the planet; taking $p'-p$ the mean motion from this, the remainder $é-e$ is the difference between the amounts of the equation. Thus, it is plain, that the difference between the mean and true motions of the planet is the rate of the increase or decrease of the amount of the equation. Therefore where the amount of the equation becomes greatest, the rate of its increase or decrease will be nothing; or the difference between the mean and true motions equals 0. But as the amount of the equation becomes greatest, when the planet reaches the point of the excentric cut by the transverse diameter of the concentric (see the note on verses 15, 16 and 17 of Chapter V), the rate of its increase or decrease must be nothing; that is, the difference between the mean and true motions will be nothing at the same point. This is the principle of the maxima and minima, with which, it is thus evident, Bháskara was acquainted.

5. He ascertained that when the arc corresponding to a given sine or cosine is found from the table of sines, this will be not far from its exact value, when it is not nearly equal to 90° or 0° respectively.†

6. He discovered the method of finding the altitude of the sun, when his declination and azimuth and the latitude of the place are given. This is a problem of Spherical Trigonometry, which he first solved by two rules in the *Gaṇitádhyáya.* Of these two rules, we have shown one in the note on verse 46 of the 13th Chapter of the *Goládhyáya,* and the other is the following :—

shown by Bháskara in his analysis, is in the highest degree remarkable; that the formula which he establishes, and his method of establishing it, bear more than a mere resemblance—they bear a strong analogy—to the corresponding process in modern mathematical astronomy; and that the majority of scientific persons will learn with surprise the existence of such a method in the writings of so distant a period, and so remote a region.' Ed.]

* [*Siddhanta-S'iromaṇi.* Chap. V, 39. Ed.]

† [*Siddhanta-S'iromaṇi.* Appendix. Ed.]

Multiply the equinoctial shadow by the radius and divide the product by the cosine of the azimuth. Assuming the result as an equinoctial shadow, find the sine of an assumed latitude, *i. e.*, finding the *Akshakarṇa* from this equinoctial shadow, say :—

as the *akshakarṇa*
: the equinoctial shadow or the result
: : the radius
: the sine of assumed latitude.

Now the sine of the sun's declination multiplied by the sine of latitude of the given place gives the sine of assumed declination.

Add the assumed declination to the assumed latitude, when the sun's declination is south ; but when the declination is north, subtract it. The result will be the zenith distance of the sun.*

Demonstration. First of all he found the shadow of the gnomon, when the sun, revolving in the equinoctial, arrived at the given vertical circle, *i. e.*, when the sun has the given azimuth, as follows :—

Draw a circle on a level surface with a given radius, and draw two diameters perpendicular to each other, east and west and north and south ; then, at the equinoctial day, if we place a gnomon of 12 digits on the level so that the end of its shadow fall on the centre, the distance of the gnomon's bottom from the east and west line must be equal to the equinoctial shadow of the given place. Now draw a line from the centre to the gnomon's bottom and produce it. It will meet the circumference at the distance of the complement of the azimuth from the east or west point.

Then say—

as the cosine of the azimuth
: the radius
: : the distance of the gnomon's bottom from the east and west line, *i. e.*, the equinoctial shadow
: the gnomon's shadow.

From this shadow find its hypothenuse, then say

as the hypothenuse
: shadow
: : radius
: the sine of the zenith distance when the sun is in the equinoctial having the same azimuth.

Call this sine the sine of assumed latitude.

Then by similar triangles—

as the sine of the latitude of the place in the plane of the meridian

* That is, assuming the given place of the observer to be in the northern hemisphere.

: the sine of the assumed latitude in the plane of the vertical
:: the sine of the sun's declination in the plane of the meridian
: the sine of the assumed declination in the plane of the vertical.

This is the sine of the arc of the vertical circle intercepted between the equinoctial and the sun's place.

Add this arc to the assumed latitude, or to the arc of the vertical circle from the zenith to the equinoctial when the declination is south; but when it is north substract the arc, the result will be the zenith distance of the sun. Hence the rule.

Then he says that if the complement of the sun's azimuth be less than his amplitude, when he is in the northern hemisphere, the vertical circle will cut the diurnal circle in two points above the horizon. Hence on the same day the sun will enter the same vertical circle at two different times, and therefore the sun's zenith distance will admit of two different values. Bháskara determined these two values thus:—

Subtract the assumed latitude above found from 180°. The remainder will be the second value of the assumed latitude. Then from these two values of the assumed latitude find the two different values of the zenith distance. The reason is very plain.

7. The ancient astronomers, Lalla, S'rípati, &c., erroneously used the versed sine and radius in finding the *valana* or variation (of the ecliptic). Bháskara himself refuted their rules variously, and used the right sine and the cosine of decliuation in the place of the versed sine and the radius respectively (see the last portion following the 29th verse of the 8th chapter of the *Goládhyáya*).

8. It is stated in the *Súryasiddhánta* and other ancient astronomical works, that the end of the gnomonical shadow revolves in the circumference of a circle, which Bháskara boldly refuted.

Besides the above Bháskara discovered many other matters which are not so important as to deserve mention here. He wrote an annotation called *Vásanábháshya* on his work himself, the style of which is very good and plain. Before he wrote this commentary, he composed two other works,—one a *Karaṇa** and the other called *Sarvatobhadra-yantra*, to find the hour of the day. Both of these works are now extant. He wrote another *Karaṇa* in the 69th year of his age, which is now very common. It appears, therefore, that Bháskara lived to the age of more than 69 years. After him, no great astronomer has appeared among the Hindús up to the present time.

* A treatise on astronomical calculation, where the epoch is taken from the commencement of the work.